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How to Value an Interest Rate Swap

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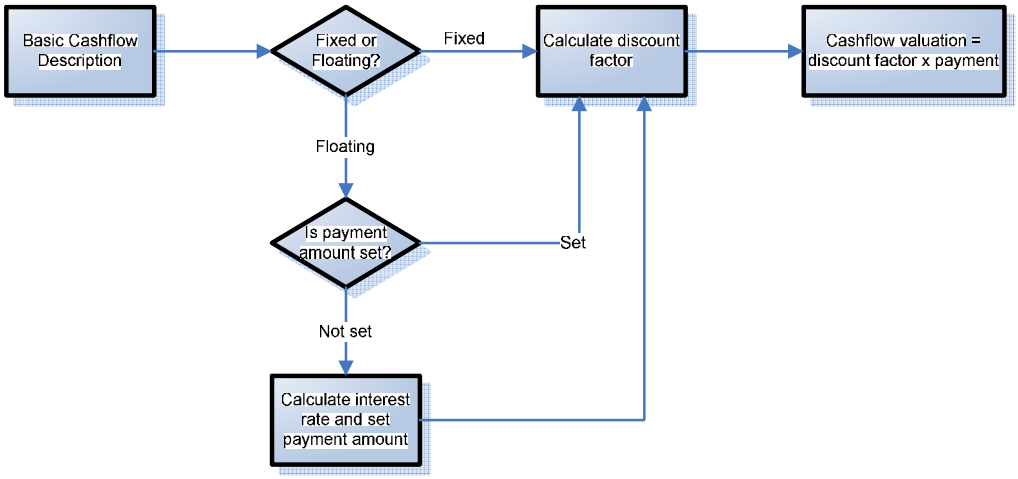
1. Introduction

The purpose of this document is to explain how to value an interest rate swap. A swap consists of a series of future cashflows to be either paid or received. Each cashflow must be valued individually. Although in theory it is not necessary to calculate and value all cashflows but instead use approximations, in practice it is necessary to find all payment dates and calculate the payments to be made and received on each. To value each cashflow there are various pieces of data that we must either be given or calculated. Essentially we need to know how much money is to be paid or received and at what date in the future. This latter piece of information allows us to calculate how much the payment must be discounted by to find its present value. The simplified process is show below.

Cashflow Valuation Overview



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The amount to be paid is either a fixed amount or may be related to future interest rates. In either case the payment will be related to some notional principal and will represent a proportion of that principal. The precise size of the payment will depend on the interest rate for the swap and the number of days the payment covers. The method for finding it is described below.

Finding a fixed rate or a set floating rate cashflow

Given a swap with a notional amount of £1,000,000 any future cashflow depends on the accrual period for that cashflow, that is how many days it is a payment for, and the interest rate which applies for that cashflow. If we assume a 90 day accrual period and a 30/360 daycount and initially consider a fixed payment with a fixed interest rate of 5% the cashflow will be

£1,000,000 x 5% x 90/360 = £12500.

For a floating rate payment where the accrual period has already started the interest rate will have been set and the calculation will be identical to the one above.

The trickier case is where the start of the accrual period is in the future so no rate has yet been set. In this case a rate must be calculated and the method for doing this is described below.

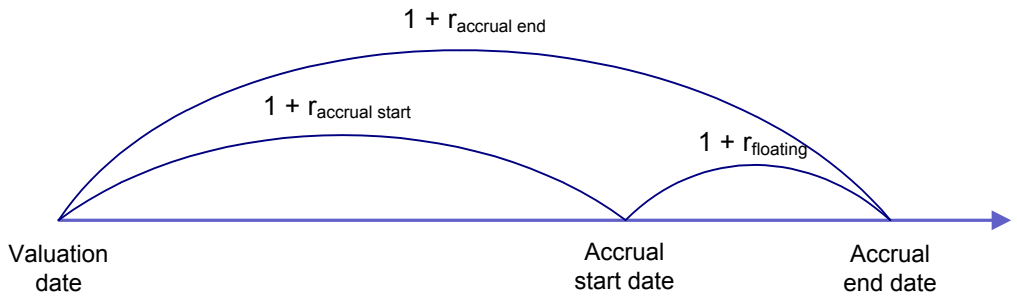
Once we ascertain these facts, if we know something about prevailing interest rates in the currency of the swap we can calculate the value now of the known payment to be received at a known date in the future. The process of putting a present value on a future cashflow is called discounting.

Inevitably, not all the data we need are readily available and calculations are required along the way. The date of the payment must be calculated taking the daycount of the swap into account and also when weekends and holidays lie since payments must be made on business days. The size of the cashflow may or may not be known. Payments can be fixed, where an interest rate is set at the inception of the trade, or floating, where the interest rate depends on prevailing interest rates during the life of the swap and is set at the start of the accrual period.

Amongst the floating payments we will come across there are those where an interest rate has been set because the accrual period has started and the setting date passed and those where the interest rate is unknown because the setting date remains in the future. In the latter case we must use interest rate predictions to calculate an estimated interest rate for the setting date. This will almost certainly involve interpolating between future rates. Interpolation is another process which will appear in various guises in the method and is discussed in the next section.

Finding an unset floating rate cashflow

Floating cashflows where the interest rate is not set are the trickiest cashflows to find. They occur on the floating side of a swap when the accrual period for the cashflow has not started so the interest rate for the period is not yet known. In this case we need to know the interest rate from the valuation date to the start of the accrual period and the interest rate from the valuation date to the end of the accrual period.



$$1 + r_{accrual\ end} = (1 + r_{accrual\ start}) \times (1 + r_{floating})$$

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The rate we need to find is the one which applies between the accrual start date and the accrual end date, referred to as $r_{floating}$. By looking at the diagram we can see that the rate from the valuation date to the end date is $(1+r_{accrualend})$, but this must, by definition, also be equal to $(1 + r_{accrualstart})$x$(1+r_{floating})$, otherwise by arbitrage it would be possible to profit on the anomaly. Therefore

$$r_{floating} = \frac{(1 + r_{accrualend})}{(1 + r_{accrualstart})} - 1$$

The payment is then calculated in the same way as described in the previous box. The only complication is that we may not know the rate for the accrual start and end days and these must be found by interpolation as discussed below.

So to reiterate, to value a given cashflow the two pieces of information that are required are the size of the future payment and the factor this must be multiplied by to find its present value. The approach for finding the so called 'discount factor' is discussed next.

So what is a discount factor? The following scenario illustrates the process of discounting and finding discount factors

Discounting and Discount Factors

If we invest £100 for 180 days when the interest rate for the next 180 days is 3% then at the end of the period our investment would be worth

£100 x (1 + 3%) = £103.

This is the future value of the £100. Conversely, if we knew we were to receive £100 in 180 days and the same interest rate applied we could calculate its value today, or present value, by **discounting** as

£100 ÷ (1 + 3%) = £97.09.

The **discount factor** in the above scenario is the number the £100 to be received in 180 days must be multiplied by to find its present value. We can see it is equal to 1 ÷ (1 + 3%) = 0.9709. Discount factors are often used instead of interest rates since they contain information about the rate for the given period whereas rates are often quoted as annualised figures.

So, once we know the size of the payment and the discount factor the value of the cashflow is easily found as discussed above. The value of a swap is then found by summing the values of all the cashflows.

Interpolation of Discount Factors

In the scenario above, suppose we do not know the interest rates or discount factors for the next 180 days but instead know the interest rates for the next 120 days and the next 360 days to be 3% and 5% respectively. These rates correspond to discount factors of 0.9709 and 1÷ (1 + 5%) = 0.9523. The discount factor for the next 180 days must then be found by **geometric interpolation** between the two given discount factors as follows:

$$\Gamma = \frac{T_{after} - T_{unknown}}{T_{after} - T_{before}} \quad \text{and} \quad DF_{unknown} = DF_{before}^{\Gamma(T_{unknown}/T_{before})} \times DF_{after}^{(1-\Gamma)(T_{unknown}/T_{after})}$$

Where Γ is a factor which contains information about the location of the payment day within the interval and the *DF*s are discount factors.

In the case under consideration T_{before} = 120, $T_{unknown}$ = 180 and T_{after} = 360 so Γ=0.75 and the discount factor we are looking for is given by,

$$DF_{unknown} = 0.9709^{0.75(180/120)} \times 0.9523^{0.25(180/360)} = 0.9614$$

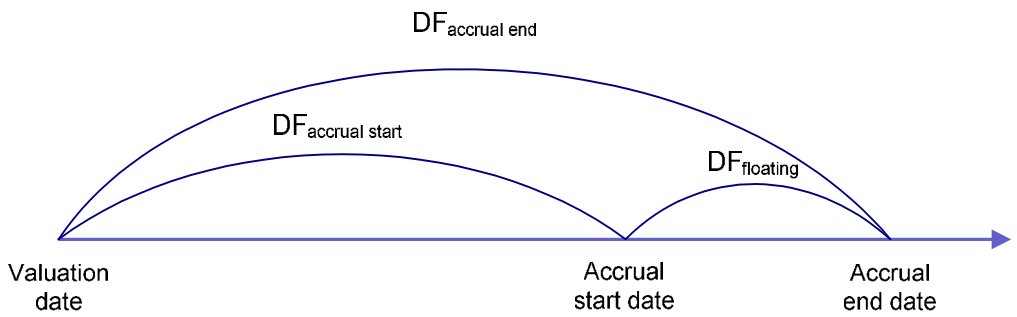
This can be seen to be a reasonable result since in value it lies between the two known discount factors.

Interpolation of discount factors also lies behind the method for finding unset floating rates as described below

Finding a Forward Rate

Say a floating rate cashflow is to have its interest rate set in 120 days, that is the accrual starts in 120 days, and assume the accrual period lasts for 60 days and the daycount is ACT/360. We wish to find the rate for that 60 day period, this is known as the **forward rate**.

If we wish to value this cashflow we must first, by interpolation if necessary, find the discount rates for the accrual start date and the accrual end date, as discussed above. By the same principle as described in the boxed text 'Finding an unset floating rate cashflow' we can say the following:



$$DF_{accrual\ end} = DF_{accrual\ start} \times DF_{floating}$$

By definition this relationship must be true and hence

$$DF_{floating} = \frac{DF_{accrualend}}{DF_{accrualstart}}$$

However, to find the unknown cashflow we know we must multiply the notional value of the swap by the interest rate, not a discount factor. We know that for a known rate, *r*, the relevant discount factor, *DF*, is given by *DF = 1/(1 + r).* However *r* here is the actual rate for that period and we need to find an annualised interest rate so must also take into account the day count. If the days in the accrual period is $d_{accrual}$ and in the year is d_{year} then

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$$r_{floating} = (\frac{1}{DF_{floating}} - 1) \times \frac{d_{year}}{d_{accrual}}$$

In the example we are considering we already know the discount factors for the start and the end to be 0.9709 and 0.9614 respectively. Therefore

$$DF_{floating} = \frac{0.9614}{0.9709} = 0.9902 \text{ and so } r_{floating} = (\frac{1}{0.9902} - 1) \times \frac{360}{60} = 5.93\%$$

As an example we will now work through the valuation of a single cashflow. Since the most difficult case is the unset floating rate cashflow we will consider this. The data describing the cashflow are given below, along with a table of discount factors to be used for the valuation.

Category	Value
Payment date	31/5/2011
Currency	US dollars
Leg type	floating
Spread	0
Notional amount	99000000
Accrual days	92
Day count fraction	ACT/360
Accrual start date	28/2/2011
Accrual end date	31/5/2011

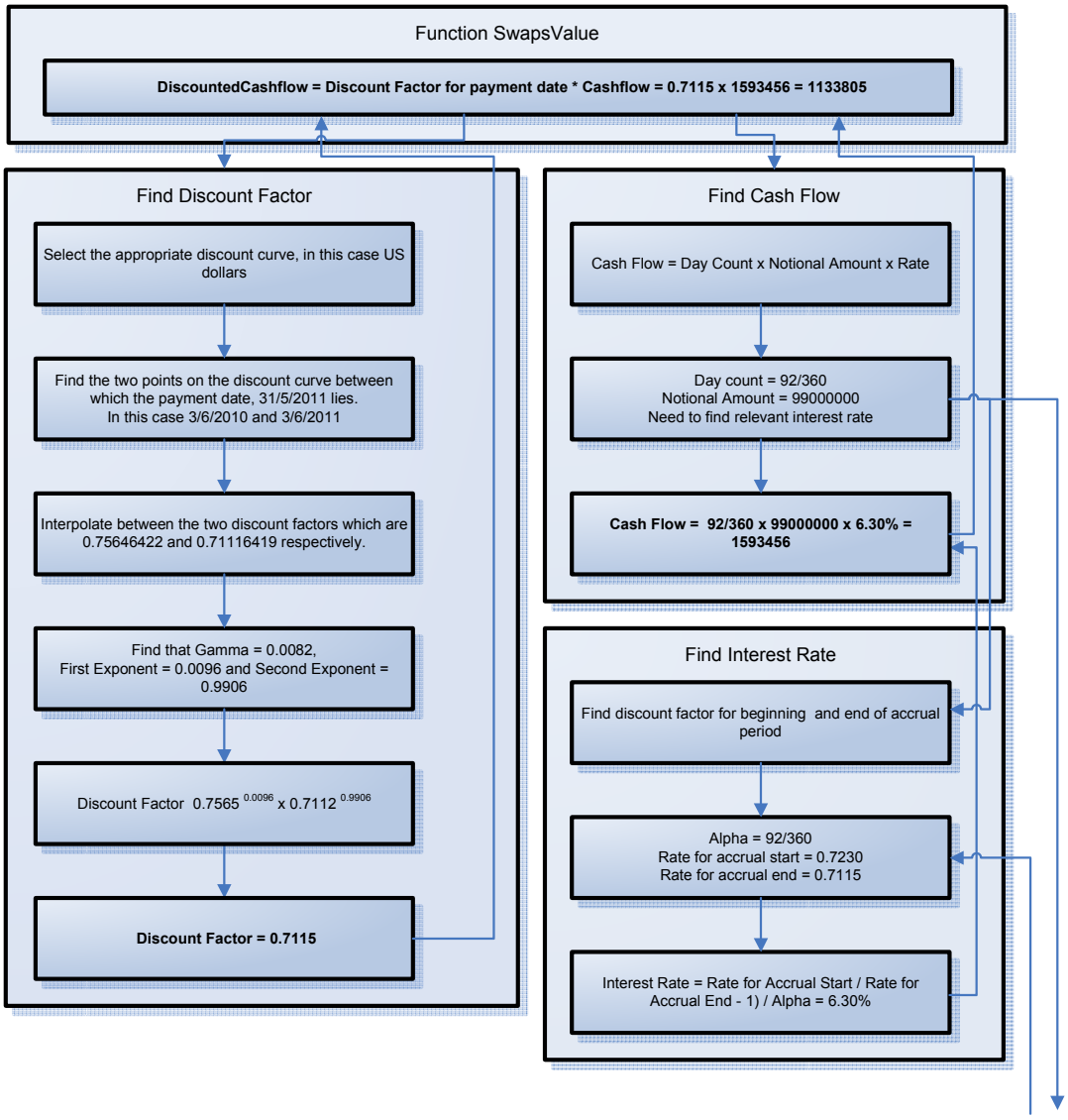
Date	Yield	Discount Factor
10/06/2004	1.05%	0.999742
06/07/2004	1.15%	0.99890847
03/08/2004	1.26%	0.99784882
03/09/2004	1.38%	0.99648303
03/12/2004	1.66%	0.99168193
03/03/2005	1.93%	0.98571557
03/06/2005	2.19%	0.97846003
02/06/2004	1.08%	0.99997056
05/12/2005	2.66%	0.96107188
05/06/2006	3.03%	0.94169295
04/06/2007	3.65%	0.89769291
03/06/2008	4.11%	0.85093172
03/06/2009	4.47%	0.80335474
03/06/2010	4.75%	0.75646422
03/06/2011	4.98%	0.71116419
04/06/2012	5.17%	0.66774085
03/06/2013	5.33%	0.62634898
03/06/2014	5.47%	0.58689432
03/06/2016	5.70%	0.51398236
05/06/2017	5.79%	0.48076473
04/06/2018	5.87%	0.4496552
03/06/2019	5.94%	0.42020787
03/06/2024	6.15%	0.30262602

04/06/2029	6.18%	0.22303292
05/06/2034	6.17%	0.16548816

The valuation involves two processes as described in the diagram below. Finding the size of the cashflow and finding the discount factor which corresponds to the payment date. To find the cashflow it is necessary to pin down an interest rate for the accrual period which is done by finding discount factors for the start and end of that period and proceeding as described in the boxed text above.

The process leads to a valuation of $1133805.

Valuing a floating cashflow where the rate is unknown



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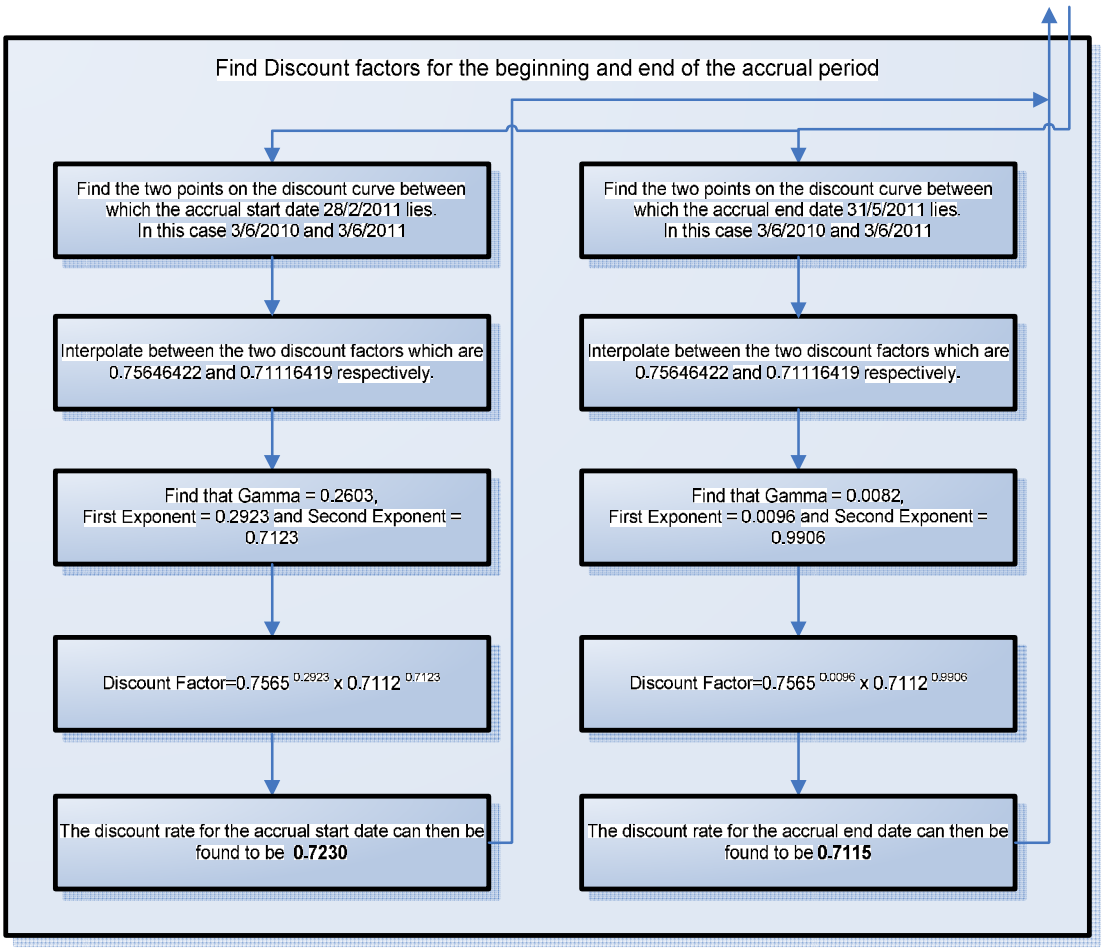
Find Discount factors for the beginning and end of the accrual period
Find the two points on the discount curve between which the accrual start date 28/2/2011 lies. In this case 3/6/2010 and 3/6/2011
Interpolate between the two discount factors which are 0.75646422 and 0.71116419 respectively.
Find that Gamma = 0.2603, First Exponent = 0.2923 and Second Exponent = 0.7123
Discount Factor=0.7565 0.2923 x 0.7112 0.7123
The discount rate for the accrual start date can then be found to be 0.7230
Find the two points on the discount curve between which the accrual end date 31/5/2011 lies. In this case 3/6/2010 and 3/6/2011
Interpolate between the two discount factors which are 0.75646422 and 0.71116419 respectively.
Find that Gamma = 0.0082, First Exponent = 0.0096 and Second Exponent = 0.9906
Discount Factor=0.7565 0.0096 x 0.7112 0.9906
The discount rate for the accrual end date can then be found to be 0.7115